

MULTICONTINUUM FAILURE ANALYSIS OF COMPOSITES

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SUMMARY

A fundamental challenge encountered in progressive failure analysis of composite structures is to *efficiently* cross multiple geometric scales to capture microstructural information where failure initiates while recognizing the practical constraints imposed by the size of the structure. In the progressive failure analysis presented here-in, nonlinear finite element analysis is used to bring stress/strain resolution from the laminate level down to the individual laminae. Embedded in the finite element analysis is a multicontinuum theory (MCT) algorithm that further decomposes lamina stress/strain fields down to the fiber and matrix constituent level stress/strain fields. Individual failure criteria for the fiber and matrix constituents are then developed based on their respective constituent stress fields.

Motivated by the results of the World-Wide Failure Exercise [1-4], the present work is aimed at improving aspects of the original MCT failure analysis developed for the Exercise. Notably, a revised matrix failure criterion is proposed for glass/epoxy composites. Moreover, the post matrix-failure response of composite laminates is altered to allow for gradual load redistribution in multidirectional laminates.

The revised failure analysis is applied to a variety of failure predictions that bring out various important aspects of the current state of MCT. Problems addressed include the effects of hydrostatic stresses on failure envelopes, a study of thermally induced stresses (including cure stresses) and their effect on thermo-mechanical failure, structural analysis of progressive failure of cruciform specimens, and compressive failure of multidirectional laminates.

Keywords: failure analysis, composites, multicontinuum

INTRODUCTION

Virtually all successful failure theories developed for composite laminates recognize that different failure criteria apply for the fiber and matrix within a composite material. Hashin [5] was one of the pioneers of such an approach when he proposed failure criteria for both the fiber and matrix materials within a composite based on the *composite* stress fields. In the spirit of Hashin, the multicontinuum theory analysis presented here-in represents a progressive failure analysis based on independent failure modes of the fiber and matrix constituents. However, rather than utilize *composite* stresses to predict *constituent* failure, MCT utilizes *constituent* stresses to predict *constituent* failure of the fiber and matrix. Continuum level constituent information may be generated in a numerically efficient manner utilizing a multiscale decomposition originally developed by Hill [6].

This paper presents a revised progressive failure analysis based on the original MCT failure analysis developed by Mayes and Hansen [7]. The analysis presented here assumes linear elastic behavior of the fibers up to fiber failure and inelastic behavior of the matrix caused by a variety of deformation mechanisms forming sequentially from submicrocrack

(void) accumulation, to macroscopic cracks such as transverse ply cracks (parallel to the fibers), and finally, to complete matrix failure characterized by matrix crack saturation.

In the case of the matrix material, damage may initiate at loads well below ultimate matrix strengths. The degree of damage is tracked from initiation through ultimate matrix failure with material properties continuously modified as damage accumulates. An important observation that is critical to the numerical efficiency of the analysis is that the relationships between composite properties and damaged constituent properties are completely determined *prior* to structural analysis. This feature distinguishes the MCT approach from other advanced progressive failure schemes that incorporate constituent level effects using a coupled nonlinear structural/micromechanics approach.

OVERVIEW OF MCT

The fundamental premise underlying continuum mechanics is that all mathematical variables represent average values of the quantity of interest. The averaging occurs over a material volume whose physical dimensions are small compared to the physical dimensions of the system of interest, yet large enough to represent the microstructure of a “material point.” The latter requirement is of particular interest in the context of the analysis of composite materials. Specifically, in the case of a continuous fiber unidirectional composite material, a material point must represent the characteristics of a volume large enough to contain numerous fibers and the surrounding matrix as shown in Figure 1(a). The macroscopic value used to characterize the stress tensor at a point in the continuum is given

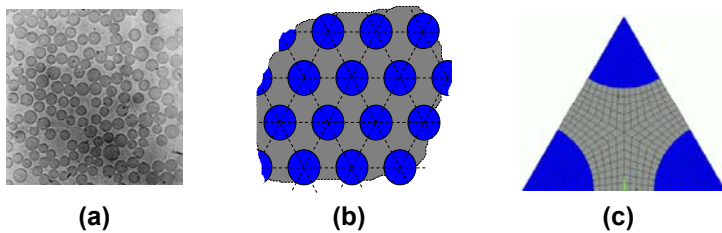


Figure 1. Progression of micromechanical modeling of a unidirectional composite material showing: (a) an actual micrograph with random fibers, (b) an idealized hexagonal packing of the fibers, and (c) a repeating unit cell for hexagonal packing with a finite element mesh.

more, clearly identifiable constituents with drastically different material properties. Hence, the composite material of Figure 1(a) may be viewed as two interacting continua composed of fibers (*f*) and matrix (*m*), respectively.

Given a composite stress and strain field, and assuming linear elastic or linear viscoelastic behavior in an incremental loading scheme, it is a straightforward matter to decompose the composite fields into their constituent level stress/strain fields for the fiber and the matrix. The specific linear elastic equations applicable to this decomposition may be found in Mayes and Hansen [7]. For brevity, the details have been omitted here-in.

For thermo-mechanical problems, a relation between composite and constituent coefficients of thermal expansion must also be established. The link between composite and constituent thermo-mechanical properties is accomplished using a finite element micromechanics model. Figure 1 shows a progression of modeling a unidirectional composite by idealizing the microstructure and eventually developing a finite element model of a representative volume element. As an aside, we note that one may choose to develop composite properties using a fully random packed representative volume element to represent the microstructure of Figure 1(a).

by the volume average of the micro-stress field in the domain represented in Figure 1(a).

The concept of a multicontinuum simply extends the notion of a

continuum to reflect coexisting materials within a material point. Such an extension is natural in any case where there are two, or

By utilizing an MCT analysis, we are afforded the opportunity to access constituent level information in a full-scale structural analysis, opening a unique window into constituent material behavior. Constituent level stress information can provide dramatic insights into composite material failure that are simply unattainable with a conventional structural analysis. For instance, in the case of cryogenic cooling of multidirectional graphite/epoxy composite laminates, thermally induced matrix stresses may be large enough to cause matrix cracking. Moreover, the ply level residual stress fields caused by mismatched thermal expansions of individual laminae play a minor role compared to the mismatched thermal expansion properties of the fiber and matrix within a lamina [8]. The constituent level thermal stresses that play such a significant role in matrix cracking can only be captured at the structural level using an MCT analysis.

MCT FAILURE ANALYSIS

In this section, we provide the details behind the substantive changes made to the MCT progressive failure analysis presented by Mayes and Hansen [7] for the first World Wide Failure Exercise (WWFE). We emphasize that the fundamental nature of the approach, reflected by a computationally efficient, multiscale finite element analysis, is unaltered.

Reexamining Constituent Failure Criteria

The current constituent level failure criteria are based on revised failure criteria originally proposed by Mayes[7]. Simple quadratic stress-based failure criteria (Eqs. 1-2) expressed in terms of transversely isotropic stress invariants of the constituents (Eqs. 3) were used to predict constituent failure within a lamina.

$$\pm A_{1f} I_{1f}^2 + A_{4f} I_{4f} = 1, \quad (1)$$

$$\pm A_{1m} I_{1m}^2 - \pm A_{2m} I_{2m}^2 + A_{3m} I_{3m} + A_{4m} I_{4m} - \pm A_{5m} I_{1m} I_{2m} = 1, \quad (2)$$

where

$$\begin{aligned} I_1 &= \sigma_{11}, \\ I_2 &= \sigma_{22} + \sigma_{33}, \\ I_3 &= \sigma_{22}^2 + \sigma_{33}^2 + 2\sigma_{23}^2, \\ I_4 &= \sigma_{12}^2 + \sigma_{13}^2. \end{aligned} \quad (3)$$

In Eqs. 1 and 2, the $I_{i\beta}$ terms denote transversely isotropic stress invariants for each constituent, $\beta=f$ for fiber, $\beta=m$ for matrix. The coefficients A_{if} and A_{jm} , leading the invariants, are constituent failure parameters, generally derived from experimentally determined composite ultimate strength data through correlation with the MCT decomposition. The \pm signs in these equations indicate a dependence of the parameter on tensile versus compressive stress. Details on the computation of these parameters for the analyses presented here-in may be found in Kenik [8]. With the exception of a single parameter, $\mp A_{5m}$, all values of A_{if} and A_{jm} may be determined from a standard set of uniaxial test data. Moreover, parameter $\mp A_{5m}$ may be determined from unidirectional composite biaxial test data ($\sigma_{11}-\sigma_{22}$). In the absence of biaxial data, a reasonable approximation of $\mp A_{5m}$ may be obtained based on the orthotropic failure criteria of Hill [9].

The fiber failure criterion of Eqn. 1 is unaltered from that used by Mayes [7]. In contrast, a revised matrix failure criterion is introduced in Eqn. 2 to accommodate the influence of matrix stresses in the fiber direction given by σ_{11m} . The original motivation for neglecting this stress component was that failure in the fiber direction is assumed to be

dominated by fiber failure. However, matrix stresses, σ_{11m} , can play a significant role in *matrix* failure in some multiaxial load cases of glass/epoxy composites. As a specific example, consider Figure 2 representing *Case 3* of the WWFE. The experimental data shown in quadrant IV of Figure 2 clearly indicate a loss of transverse compressive strength in the presence of axial tensile stress, σ_{11m} . Figure 2 also shows theoretical predictions of the updated MCT failure analysis based on Eqs. 1 and 2, and the original MCT predictions of Mayes and Hansen [7]. The figure indicates the revised matrix failure criterion appropriately captures the influence of axial matrix stresses.

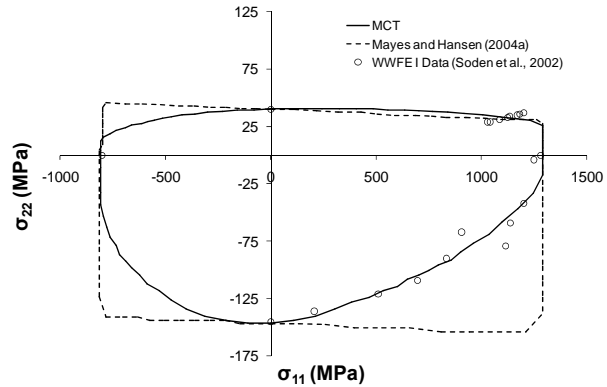


Figure 2. Original (Mayes and Hansen [7]) and revised MCT-based biaxial failure envelopes for E-Glass /MY750 compared against WWFE experimental data (Soden *et. al.* [3]).

It should be noted that the matrix stress interaction phenomenon described above generally is not observed in carbon/epoxy composites as the axial *matrix* stress is significantly smaller compared to glass/epoxy composites. As a result, the biaxial ($\sigma_{11} - \sigma_{22}$) max stress-like behavior predicted by Mayes and Hansen in Figure 2 is more typical of carbon/epoxy materials.

Matrix Cracking and the Post-Failure Response

The MCT analysis of Mayes and Hansen [7] presented in the WWFE utilized an overly conservative load redistribution scheme within a multidirectional laminate in the presence of matrix failure. In the case of a unidirectional composite, matrix failure leads directly to failure of the composite. Based on this observation, Mayes and Hansen modeled the occurrence of matrix failure by zeroing the material properties of the matrix constituent and redistributing the loads throughout the structure as well as the remaining fiber constituent. As expected, this approach worked very well for unidirectional materials.

In contrast to unidirectional composite laminates, zeroing the matrix properties at matrix failure in multidirectional laminates produced conservative results. The underlying reason is that local matrix failure in a multidirectional laminate results in substantial load redistribution to adjacent plies. This load redistribution is readily transferred back to the failed ply a short distance away from the localized matrix failure. The net effect is that the “failed ply” continues to sustain load. Knops and Bögle [10] clearly document a “laminate effect” experimentally, showing a gradual degradation in normal and shear moduli in the presence of increasing transverse crack density induced by tensile loads.

RESULTS

Several results covering a broad spectrum of thermo-mechanical loads and structural complexity are presented to bring out some of the distinguishing features of the MCT methodology. To begin, failure envelopes from the WWFE are revisited to demonstrate improved failure predictions of the updated MCT analysis. Next, the effects of large hydrostatic stresses on composite failure are examined. The results presented represent predictions taken from a contribution to the Second World-Wide Failure Exercise [11]. An analysis of compressive behavior of multidirectional laminates is also presented.

A question that continually surfaces in progressive failure theories is the ability to predict the effects of thermal loads in addition to mechanical loads. Not only are composites

routinely subjected to thermo-mechanical loads, the effect of residual stresses generated during cure cycles is of interest. Two thermo-mechanical results including modeling the cure stresses from cool down are presented.

Finally, although most results presented are based on a simple laminate analysis requiring a finite element per layer, we present results for two structural problems requiring a detailed finite element mesh. These include modeling a cruciform specimen for biaxial testing and predictions of tank leakage for a pressurized tank at cryogenic temperatures.

Weeping through the wall of a laminate

Figure 3(a) shows a complete failure envelope for $\sigma_y:\sigma_x$ loading of an E-glass/MY750/HY917/DY063 $[\pm 55^\circ]_S$ laminate. Current MCT results show substantial improvement in quadrant III compared to the results of Mayes and Hansen [7]. Moreover, the unique nature of the failure envelope provides for little data in quadrants II and IV and the analytical results support this.

Quadrant I data shows major differences in ultimate failure predictions from both Mayes and the updated MCT analysis compared to the data. It is believed the data of quadrant I is questionable in that the loading—particularly the loads with large axial stresses, σ_{yy} , produces significant shear strains resulting in finite strain effects in the form of fiber orientation changes. Any reorienting of fibers may dramatically increase strengths and ultimate failure may be driven by fiber rupture. Also, the experimental data shown in Figure 3(a) is an assembly of several tests on composite tubes as described by Soden [2,3] and may not be accurately captured using a simple laminate analysis.

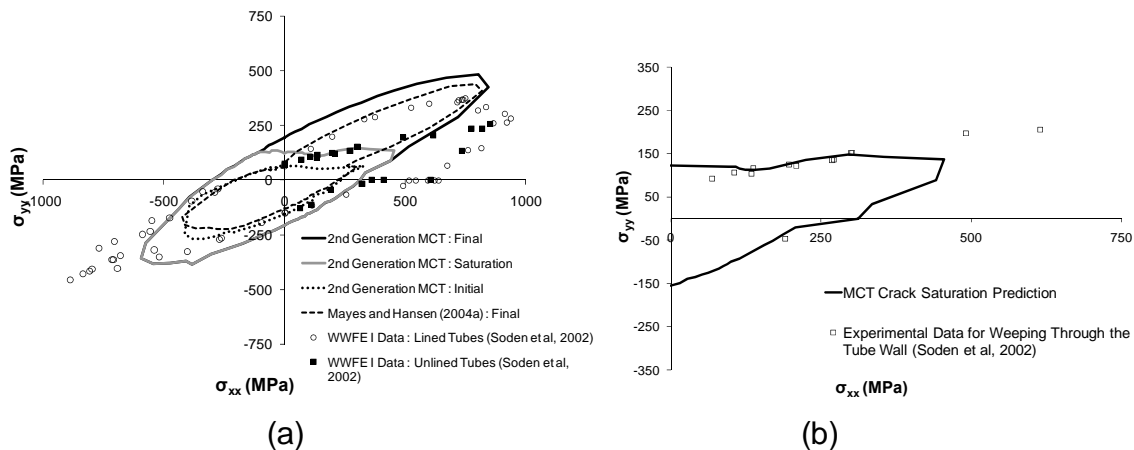


Figure 3. (a) Biaxial, $\sigma_{yy}:\sigma_{xx}$, failure envelope for a $[\pm 55^\circ]_S$ laminate made from E-glass/MY750/HT917/DY063 compared against experimental data of Soden *et. al.* [3]., (b) crack saturation predictions compared to data for through wall weeping.

To illustrate the importance of identifying the role of each constituent in the final failure of the specimen, the data plotted in Figure 3(b) is limited to leakage through the wall of an unlined specimen. Leakage requires a continuous crack path that passes through each layer of the laminate. This scenario requires a dense network of matrix cracks in each layer of the laminate to increase the likelihood of continuous crack paths. In order to identify this scenario, it is theorized that the matrix material must experience crack saturation, or very near to it, through all plies in order for this significant leakage to take place.

MCT predictions for matrix crack saturation are compared to the weeping data in Figure 3 (b). There is excellent agreement with the experimental data. Figure 3(a) also clearly demonstrates that matrix weeping occurs far below ultimate loads for the laminate. This observation supports the philosophy that multiscale information is needed to accurately

capture the behavior of a composite structure as the definition of failure will change with service conditions.

Pressure Strengthening

Figure 4 shows a predicted biaxial (σ_{12} versus σ_{22} with $\sigma_{11} = \sigma_{22} = \sigma_{33}$) failure envelope for a T300/PR319 graphite/epoxy unidirectional laminate. Failure in quadrant I is driven by a combination of matrix tensile and matrix shear stresses. Failure in quadrant II is governed by matrix shear failure. Strengthening of the matrix due to hydrostatic pressure is readily apparent as the absence of this strengthening would produce a near horizontal failure prediction at 97 MPa in quadrant II which is the pure shear value provided. MCT does not predict constituent level failure under pure hydrostatic pressure for the plot scale requested in Figure 4.

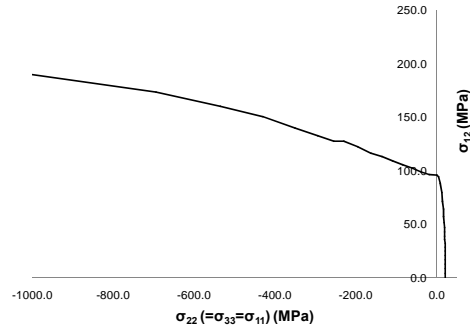


Figure 4. Case 2 of WWFE2; biaxial (σ_{12} versus σ_{22} with $\sigma_{11} = \sigma_{22} = \sigma_{33}$) failure envelope predictions for T300/PR319 graphite/epoxy unidirectional laminate.

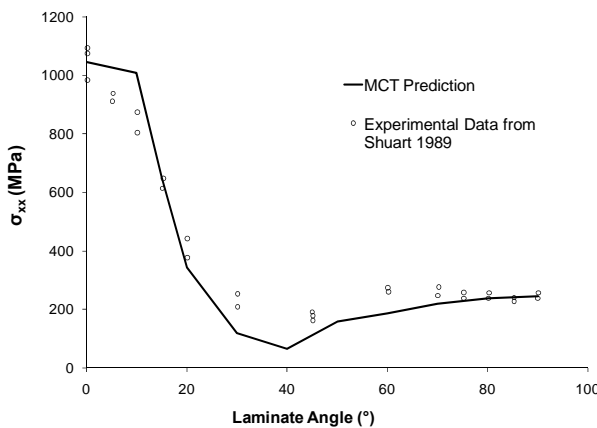


Figure 5. Compressive strength as a function of ply orientations for $[\pm\theta]$ AS4/3502 laminates from Stuart [12].

correlation with the experimental results. In particular, the experimental data shows distinct changes in the slope of the curve where the failure mode changes and MCT captures this change in failure behaviour. It is important to note that the strong correlation between MCT predictions and the experimental data is achieved using the matrix failure criterion shown in Equation 2.

Thermomechanical Loads

One of the most persistent questions that arose in the first World-Wide Failure Exercise is the impact of residual cure cycle thermal stresses on failure predictions. This question is a challenging problem to address as many of the proposed theories cannot generate constituent level thermal residual stresses in a progressive failure analysis at the structural (lamina) level. While MCT can access these self-equilibrating constituent stresses, an accurate assessment of their magnitudes requires a rather sophisticated analysis that should, at minimum, include non-isothermal viscoelastic behavior. Such an analysis is

Angle ply compression

Figure 5 shows a comparison of compressive failure of a carbon/epoxy laminate with varying ply orientations as presented in Stuart [12]. In this work, 13 different angle ply lamination schemes were tested to compressive failure. Stuart found that for $\theta < 15^\circ$, the dominate failure mode is interlaminar shear. For a lamination angle of $15^\circ < \theta < 50^\circ$, in-plane matrix shearing causes failure. Finally for $\theta > 50^\circ$, matrix compression dominates composite failure. The material properties used in the analysis are taken directly from Davila et al. [13].

The MCT results show excellent

further complicated by the need for time and temperature dependent properties of the neat resin.

In order to investigate the impact of cure stresses, a non-isothermal viscoelastic analysis of a glass/epoxy and carbon/epoxy composite was conducted with Derakane 510 as the resin material. Derakane 510 was chosen as it is a common epoxy material used in Navy composites and the material has been fully characterized under non-isothermal viscoelastic creep [14]. Following the work of Kenik [8] matrix stresses on the order of 15 MPa were generated during the cure cycle.

Based on the matrix cure stresses obtained by Kenik, failure predictions from the WWFE I were reexamined. In doing so, the ultimate strengths provided by the organizers had to be modified to account for the residual constituent stress fields. Figure 6 shows a predicted failure envelope for combined tension/shear loading of an Eglass/LY556 glass/epoxy composite, with and without residual cure stresses. The figure clearly indicates there is no change in the predicted failure envelope. Similar results were found for all other failure envelopes of unidirectional composites examined in the WWFE. Multidirectional laminates are addressed in Kenik [8]. The conclusion is that failure strengths computed at room temperature inherently account for the residual thermal stresses found in the composite. Hence, failure data of composite laminates provided at room temperature are sufficient to provide meaningful failure predictions at room temperature—to the extent of any theory’s

accuracy—without considering residual thermal stresses.

In contrast to reference temperature test data and subsequent reference temperature failure predictions, thermally induced stresses generated away from a reference temperature have a dramatic effect on failure predictions. For instance, consider failure predictions of a unidirectional T300/BSL914C carbon/epoxy laminate. Figure 7 shows a room temperature envelope along with predicted failure envelopes at $\Delta T = -150^\circ\text{C}$. The two low temperature curves show failure envelope predictions with and without cure stresses, respectively. The difference in the room temperature and low temperature envelopes is dramatic. Furthermore, the residual stresses

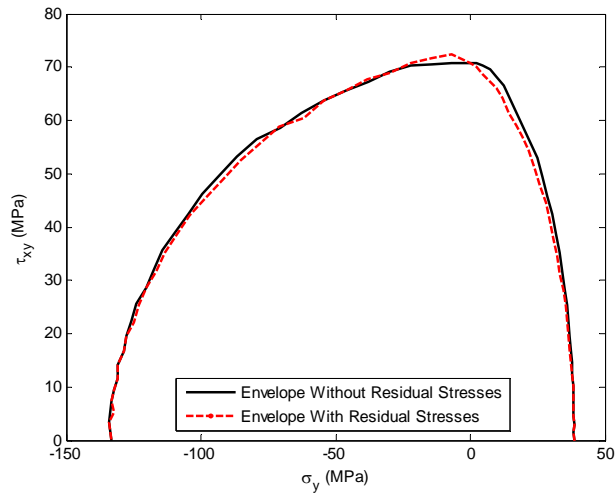


Figure 6. Comparison of the effect of residual stresses on a transverse tension/compression and shear failure envelope.

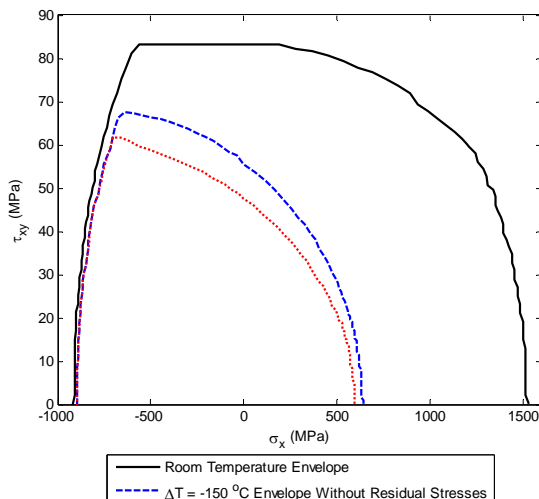


Figure 7. Comparison of the effect of cure stress on a thermo-mechanical biaxial failure envelope.

generated at the constituent level are naturally captured by the MCT. The constituent information provided by MCT opens a valuable window into failure phenomena that simply cannot be captured at the structural level with a conventional continuum theory.

Biaxially Loaded Cruciform Specimens

Welsh, Mayes, and Biskner

[15] performed a parallel

experimental and numerical failure prediction of biaxially loaded quasi-isotropic carbon composites.

The focus of this effort was to evaluate the fundamental response of a quasi-isotropic carbon composite laminate when subjected to unidirectional and biaxial loading conditions. For the numerical side of that study, the original MCT failure theory was used to predict and analyze the onset of damage and ultimate failure of the cruciform specimens. During the work presented here, the analysis performed by Welsh, Mayes, and Biskner [15] is repeated using the updated MCT failure criteria.

Figure 8 shows two analytical biaxial failure envelopes and an experimental envelope for a quasi-isotropic cruciform specimen made from IM7/977-2 carbon/epoxy. The first biaxial failure envelope was generated using a laminate analysis assuming constant stress states at each load ratio. In uniaxial stress load ratios, the strength of the laminate is predicted well. However, under multiaxial loading the predicted envelope does not correlate well with the experimental data.

The discrepancies between the laminate envelope and the experimental were suspected to be because the cruciform specimen contains a non-uniform stress state. To investigate this, a three dimension finite element model was used in the MCT progressive failure analysis. Abaqus/Standard [16] commercial finite element code was used for the analysis. Layered composite SC8R elements were used to represent one-eighth of the cruciform specimen; the model contained 3870 degrees of freedom. Model run time for the full progressive failure analysis was generally between 10 and 20 minutes.

Results generated using the three dimensional MCT progressive failure analysis show significant improvement over the laminate results supporting the hypothesis that the cruciform specimen behaves similar to a structure thus containing a non-uniform stress state. These results, coupled with reasonable run times, strengthen MCT's viability as a structural analysis tool.

Structural Analysis of an Unlined Composite Pressure Vessel

The focus of this area of study was to predict the response of an unlined composite pressure vessel under combined thermal and mechanical loading. Six 1.9L unlined composite pressure vessels were manufactured using carbon fibers and a toughened epoxy resin. The experimental test procedure began with filling the vessels with liquid nitrogen while submerged in a bath of the same liquid. Next, the vessels were pressurized with gaseous helium until either the vessel burst or significant leakage occurred.

An MCT progressive failure analysis of the composite pressure vessel was conducted using the finite element code Abaqus. A 3-D model of the vessel was developed using SC8R

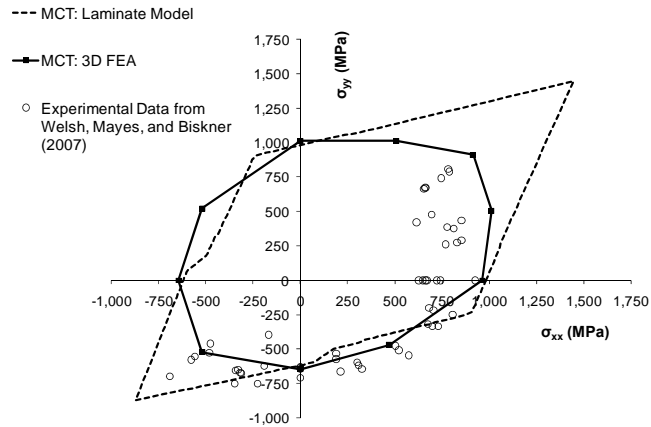


Figure 8. Experimental and MCT biaxial failure envelopes for a quasi-isotropic cruciform specimen.

elements. The procedure used for finite element analysis of the 1.9L tank was designed to simulate the experimental testing of the tank.

In the first load step (referred to as the ΔT step), the temperature of the tank was reduced from room temperature (20°C) to liquid nitrogen temperature (-196°C) in the absence of any applied mechanical loads, with appropriate node displacement constraints to prevent rigid body motion. The ΔT step did not produce any predicted localized failures in the matrix or fiber constituents. However, examination of the finite element results, revealed that the most severely stressed matrix constituent material occurred within the 90° plies of the uniform diameter cylindrical region, where the matrix failure criterion reached 27% of the value that indicates failure.

The second load step (referred to as the $\Delta T+p$ step) involves application of an internal pressure, p , while the tank remains at liquid nitrogen temperature (-196°C). To account for material and geometric nonlinearity, the pressure loading is increased in a series of equal pressure increments of .10 MPa until crack saturation is predicted. During both of the loading steps (ΔT and $\Delta T+p$), the MCT constitutive module is used to predict and track localized failures in the matrix and fiber constituents, in addition to performing stiffness degradation that is based on predicted constituent failure.

Unlike the ΔT step which did not produce any matrix or fiber constituent failure, the MCT model predicts that the $\Delta T+p$ step begins causing localized matrix constituent failure (i.e., initial matrix cracking) at a pressure loading of $p = 4.65$ MPa. This predicted initiation of matrix cracking at $p = 4.65$ MPa does not indicate that the tank will leak at this pressure. Crack saturation, as described previously in the post-failure nonlinearity section, is believed to correlate with leakage through the wall of the vessel.

For the $\Delta T+p$ step, the current MCT model predicts that matrix crack saturation occurs at an applied pressure of $p = 8.38$ MPa. This value is in excellent agreement with the observations made during pressure testing of the tanks, where the mean pressure at which leaking was observed for six tanks was 8.50 MPa. Thus, the leak pressure predicted by the crack saturation using MCT is within 2% of the mean measured leak pressure.

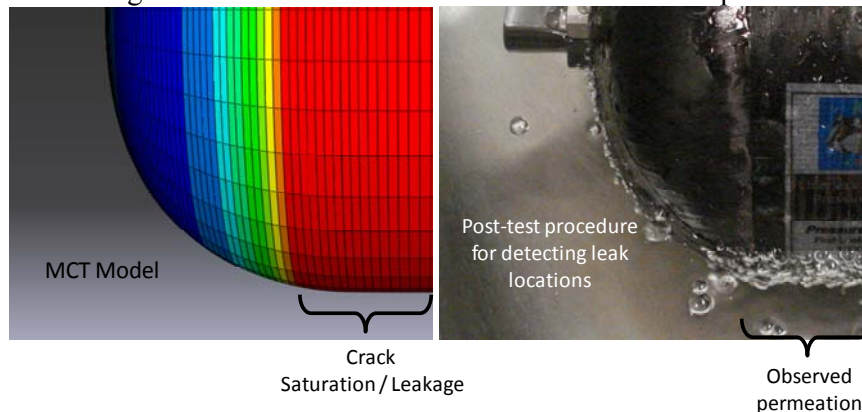


Figure 9. Predicted tank leakage compared to observed permeation on a tank after testing.

Location of crack saturation is also an important aspect of the analysis. Using the current MCT analysis, crack saturation is predicted in the constant diameter cylindrical section. Figure 9 shows the location of the MCT predicted leak location shown next to a failed vessel pressurized with air and submerged in water. The qualitative comparison of leakage location is excellent.

ACKNOWLEDGEMENTS

The research was sponsored by the Air Force Research Laboratory Space Vehicles Directorate under contract number FA 9453-07-C-0191 under the direction of Dr. Thomas Murphey. The authors are also grateful to Dr. Jeffry Welsh for invaluable discussions.

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