PHYSICS-BASED FATIGUE LIFE PREDICTION OF COMPOSITE STRUCTURES

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THEME

Composites: Multi-scale, Multi-fidelity Modelling

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SUMMARY

Widespread use of composite materials across many industries has created a need for modeling tools to accurately predict the behavior of structures comprised of these materials. In many applications, composite fatigue life is the primary factor limiting the design of a structure. However, commercial tools for predicting composite fatigue life have not been available. We present a multiscale physics-based fatigue life prediction methodology for composite structures that is computationally efficient, requires minimal fatigue characterization, and accounts for arbitrary loads and load histories. The approach has been developed to work seamlessly with structural finite element codes.

Because it is physics-bases, this method requires minimal composite characterization data. Furthermore, the method predicts fatigue life for any load history, load ratio, and loading mode. And it naturally accounts for the effects of mean stress and frequency variation on composite fatigue life. This method has been recently commercialized and is currently used to drive the fe-safe/Composites™ fatigue module. We present results using this tool where short beam shear fatigue life of a composite laminate is predicted based on lamina-level fatigue characterization.

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1: INTRODUCTION

Composite materials are becoming ubiquitous in large structures such as wind turbine blades, airframes, racecars, and watercraft. Many of the applications for which composites are now used require the composite structure to perform under cyclic loading. Thus, fatigue life prediction in composite structures is an important part of composite design, but analysis tools have not been readily available until now.

For polymer-matrix composites, fatigue failure is primarily a matrix-dominated event. Fatigue damage begins with microcrack accumulation in the polymer. These microcracks accumulate most rapidly in the early stages of fatigue life, with the accumulation rate slowing with increasing number of cycles. Ultimately, the microcracks coalesce to form a macroscopic crack that quickly causes catastrophic failure. Accurate fatigue life prediction thus requires capturing the accumulation of microcracks per cycle. The ultimate catastrophic failures typically fall into one of three categories: off-axis failure, on-axis failure, and delamination. The focus of the method described here is off-axis fatigue failure, which occurs when composite tensile loading is more than a few degrees from the fiber axis (Hashin & Rotem 1973). Ultimate failure is characterized by matrix cracking parallel to the fiber (Awerbuch & Hahn 1981; Petermann & Plumtree 2001) or by debonding between the fiber and matrix interface. In the case of a composite laminate, delamination is possible. We consider delamination as a final event so that results from accumulated matrix microcracking, thus accounting for the accumulation of matrix microcracks is sufficient for predicting fatigue failure due to delamination[(i)].

In this paper, we outline a methodology in which, composite fatigue is modeled as a matrix phenomenon using matrix-specific physics. After outlining this approach, it is used to predict short beam shear fatigue life using only interlaminar tension fatigue data for calibration, demonstrating the appropriateness of the applied physics.

2: MODEL DEVELOPMENT

(a) Overview

Structural level, physics-based modelling of composite fatigue requires three separate modeling processes: a multiscale model to link composite stresses and strains to matrix stresses and strains, a physics-based model to predict fatigue damage of the matrix material, and a link between the physics-based model and the macroscopic fatigue failure of the composite. A portion of these modeling

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[i] In the case where delaminations are present *before* initiation of fatigue loading, built-in FEA tools for crack propagation should be used rather than Helius:Fatigue.

efforts have been described by Fertig (2009). Figure 1 shows a high-level overview of the information flow in this approach. First, a finite element (FE) model is used to compute composite stresses at the peak load P_{ref} of a fatigue cycle. The element stresses from the FE model results are extracted for use in the physics-based methodology described in this paper, which is encapsulated in the software module, fe-safe/Composites™. As shown in Figure 1, the only required inputs to the model are composite strengths, the elastic properties of the composite and its constituents, fatigue material properties extracted from a single calibration S-N curve, and the load history *P(t)*.

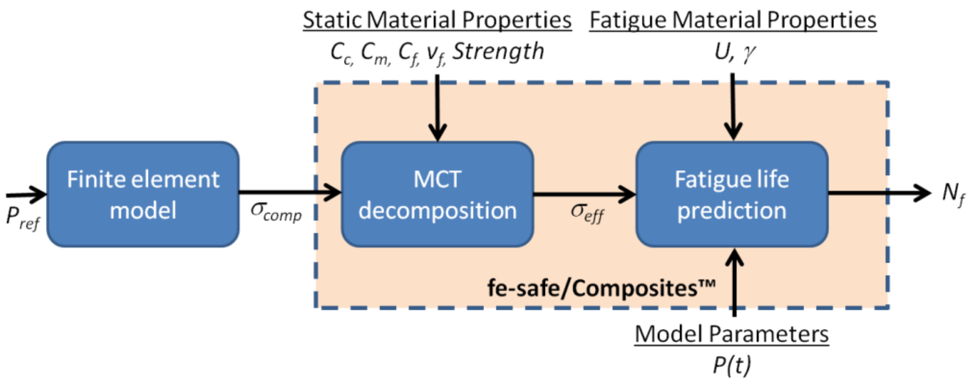

Figure 1. Schematic showing the processes involved in predicting composite fatigue failure using fe-safe/Composites™.

The multiscale modeling approach employed for linking composite stress/strain behavior with constituent stress/strain behavior is multicontinuum theory (MCT). Briefly, MCT provides an exact mapping of volume average composite stresses/strains to volume average constituent stresses/strains. So any arbitrarily complex composite stress state can be mapped to its corresponding matrix stress state. This matrix stress is converted to an effective matrix stress that can be used in an appropriate physics-based model.

The relevant physics describing the fatigue of a polymer matrix is the kinetic theory of fracture (KTF) (Regel & Tamuzh 1977; Regel' et al. 1972; Kireenko et al. 1971; Sauer & Richardson 1980; Coleman 1956), which treats fracture as a thermally activated process. We modify the baseline equations of KTF in order to accurately model the effect of different stress ratios on fatigue loading.

Using KTF in conjunction with MCT gives a rate of microcrack accumulation in the matrix. The remaining effort is to link the bond breaking rate with a macroscopic measurement of composite damage. We extend work of Hansen and Baker-Jarvis (1990) and use a damage parameter *n* that describes microcrack density accumulated as a percentage of the microcrack density at failure.

These steps are outlined in the subsections below.

(b) Multicontinuum Theory

Multicontinuum theory, as developed for two-constituent composite materials by Garnich and Hansen (1997a, 1997b), provides an elegant and computationally efficient method to extract volume-averaged stresses of the matrix and fiber from the composite stress state. Here we are interested in matrix-dominated fatigue failure, so the average matrix stress is the physically relevant parameter. The exact value of average stress in the matrix $\boldsymbol{\sigma}_\mathbf{m}$ can be written as

$$\boldsymbol{\sigma}_\mathbf{m} = \mathbf{Q}_\mathbf{m}\boldsymbol{\sigma}_\mathbf{c} - \boldsymbol{\psi}_\mathbf{m}(\Delta T) \tag{1}$$

where

$$\begin{aligned}
\mathbf{Q}_\mathbf{m} &= \mathbf{C}_\mathbf{m}\left\{\mathbf{C}_\mathbf{c}\left(\phi_m\mathbf{I} + \phi_f\mathbf{A}\right)\right\}^{-1} \\
\boldsymbol{\psi}_\mathbf{m} &= \mathbf{C}_\mathbf{m}\left\{\phi_f\left[\left(\mathbf{C}_\mathbf{c} - \mathbf{C}_\mathbf{f}\right)\left(\phi_m\mathbf{I} + \phi_f\mathbf{A}\right)\right]^{-1}\mathbf{a} + \boldsymbol{\eta}_\mathbf{m} - \left(\phi_m\mathbf{I} + \phi_f\mathbf{A}\right)^{-1}\boldsymbol{\eta}_\mathbf{c}\right\} \\
\mathbf{A} &= -\frac{\phi_m}{\phi_f}\left(\mathbf{C}_\mathbf{c} - \mathbf{C}_\mathbf{f}\right)^{-1}\left(\mathbf{C}_\mathbf{c} - \mathbf{C}_\mathbf{m}\right) \\
\mathbf{a} &= \mathbf{C}_\mathbf{c}\boldsymbol{\eta}_\mathbf{c} - \phi_f\mathbf{C}_\mathbf{f}\boldsymbol{\eta}_\mathbf{f} - \phi_m\mathbf{C}_\mathbf{m}\boldsymbol{\eta}_\mathbf{m}
\end{aligned} \quad . \tag{2}$$

In Equations (1) and (2) boldface variables indicate non-scalar quantities, $\boldsymbol{\sigma}_\mathbf{c}$ is the six-component composite stress vector; ΔT is the temperature change from the stress-free state; $\mathbf{C}_\mathbf{i}$ ($i = c, f, m$) are the reduced stiffness matrices for composite, fiber, and matrix, respectively; ϕ_f and ϕ_m are the fiber and matrix volume fractions, respectively; and $\boldsymbol{\eta}_\mathbf{i}$ ($i = c, f, m$) are the thermal expansion coefficients of the composite, fiber, and matrix, written as six-component strain vectors where the shear components are zero.

In order for the matrix stress obtained in Equation (1) to be used in the kinetic theory of fracture, it must be mapped to an effective stress. This is procedure has been described elsewhere (Fertig 2009). Essentially, the strength-life equal rank assumption is employed such that a failure criterion that utilizes matrix-averaged stresses is converted to an effective stress

$$\sigma_{eff} = \sqrt{A_t\left\{I_{m,t}\right\}^2 + \sigma_{m,12}^2 + \sigma_{m,13}^2 + A_S\left(\tfrac{1}{4}\left(\sigma_{m,22} - \sigma_{m,33}\right)^2 + \sigma_{m,23}^2\right)}, \tag{3}$$

where A_S and A_t are functions of static failure coefficients, $\sigma_{m,ij}$ is the *ij*-component of the matrix stress tensor, and I_t is the maximum matrix tensile stress normal to the fiber direction. This effective matrix stress is used to drive the physics-based equations described below.

(c) Kinetic Theory of Fracture

The kinetic theory of fracture (KTF) describes the process of bond breaking via thermally activated processes. Our composite fatigue methodology uses KTF to predict fatigue failure in the matrix constituent, which translates to composite failure. KTF for polymers has been described in detail in published literature (Regel & Tamuzh 1977; Kireenko et al. 1971; Regel' et al. 1975; Regel 1971; Coleman 1956; Zhurkov 1965). We briefly outline its development here.

All atoms and molecules at temperatures greater than absolute zero oscillate with a frequency proportional to $\frac{kT}{h}$ given by Planck's Law, where h is Planck's constant and kT is the thermal energy, described by the product of the Boltzmann constant k and the absolute temperature T. But because the thermal energy associated with oscillation is not a single number but rather a distribution, there is always a statistical likelihood that any given oscillation will have sufficient energy to overcome an energy barrier U to move from one equilibrium state to another. The likelihood of this occurring for any oscillation is given by the familiar exponential form $\exp\left(-\frac{U}{kT}\right)$. Thus, the rate of any thermally activated process can be written simply as

$$K = \frac{kT}{h}\exp\left(-\frac{U}{kT}\right). \tag{4}$$

For the purposes of fracture and fatigue, one equilibrium state is the un-microcracked state and the other is the microcracked state. The process is illustrated schematically in Figure 2 (left).

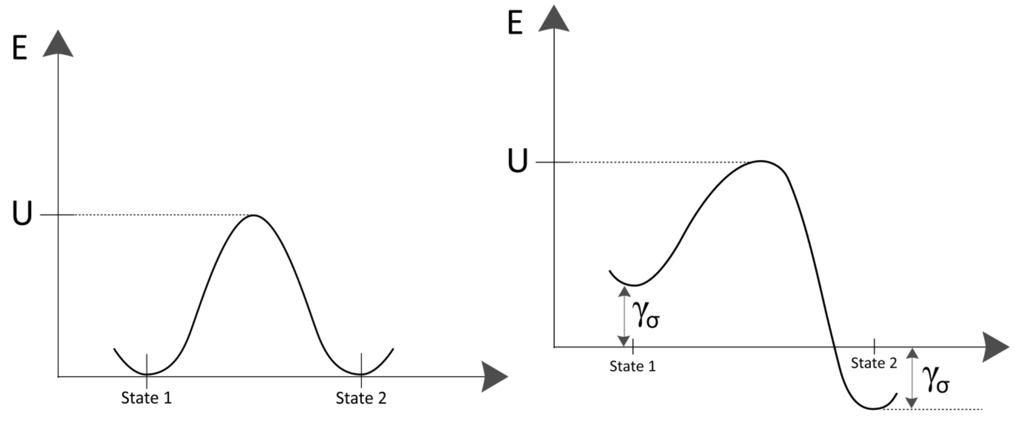

Figure 2: (Left) Energy barrier with activation energy U for transition from State 1 to State 2. (Right) Energy barrier with activation energy U-γσ for transition from State 1 to State 2.

Now consider the effect of an applied stress σ on the bond breaking rate described in Equation (4). In this context it is more helpful to think of stress in terms of energy per unit volume rather than force per unit area. Thus, if the stress is acting to facilitate the bond-breaking process it should reduce the energy barrier to bond-breaking. The amount of this reduction depends on the volume of material *γ,* the activation volume[(ii)], over which the process occurs so that the reduction in the activation energy is simply the product of activation volume and the stress. Figure 2 (right) shows how an applied stress modifies the activation energy. Taking the activated process to be microcrack accumulation and modifying Equation (4) to reflect the effect of an applied time-varying stress gives

$$K_b(t) = \frac{kT}{h}\exp\left(-\frac{U - \gamma\sigma(t)}{kT}\right), \tag{5}$$

which is the baseline KTF equation for predicting composite fatigue. Equation (3) is used to map the average matrix stress tensor into an effective stress that is used in Equation (5).

We now turn to the problem of how to account for oscillating stress. The nomenclature of stress ratio *R* and mean stress σ_m are typically introduced to describe simple sinusoidal loadings that vary from some minimum stress σ_{min} to some maximum stress σ_{max}, where

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \qquad \sigma_m = \frac{\sigma_{\min} + \sigma_{\max}}{2}. \tag{6}$$

Consider the load histories shown in Figure 3 for *R* = 0.1 and *R* = 0.6. It can be shown by integrating Equation (7) that for the same σ_{max} the average bond breaking rate $\langle K_b \rangle$ is greater for R = 0.6 than for R = 0.1. Thus KTF as presented in Equation (5) predicts that increasing the mean stress by increasing σ_{min} would increase the mean bond breaking rate for a fixed σ_{max}, which would result in a shortening of the fatigue life with increasing value of *R*. But experiments on unidirectional composites have shown that an increase in σ_{min} causes an *increase* in the fatigue life (Kawai & Suda 2004). Thus, the oscillatory nature of the stress presents an additional physical phenomenon that is not present under constant load.

The additional physical feature is the temperature change in the polymer during cycling. Experiments on pure polymers have shown that significant heating of the polymer occurs with an increase in the oscillating stress amplitude (Sauer & Richardson 1980; Kireenko et al. 1971) or an increase in the frequency (Sauer et al. 1977). In order for kinetic theory to properly predict fatigue life for any load history, this temperature increase must be accounted for. For a

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[ii] The precise definition of activation volume is somewhat ambiguous for the process of polymer fracture because it is much larger than, say, the atomic volume. Thus, it is typically only defined as the volume over which the activated process occurs.

sinusoidal cycle, the rate of energy dissipated per second $\dot{E}$ is given by (Sauer & Richardson 1980):

$$\dot{E} = \pi f J'' \sigma_a^2 , \tag{7}$$

where f is the oscillation frequency, J'' is the loss compliance, and σ_a is the stress amplitude. We assume that the temperature increase is proportional to the energy dissipated. The temperature T used in Equation (5) is calculated from the nominal temperature T^* such that

$$T = T^* + \psi \sum_{i=1}^{n} \frac{\left(\Delta\sigma_{eff}\right)^2}{\Delta t} , \tag{8}$$

where ψ is a constant of proportionality, $\Delta\sigma_{eff}$ is the magnitude of effective stress change over a span of time Δt, and n is the number of different stress ranges per cycle.

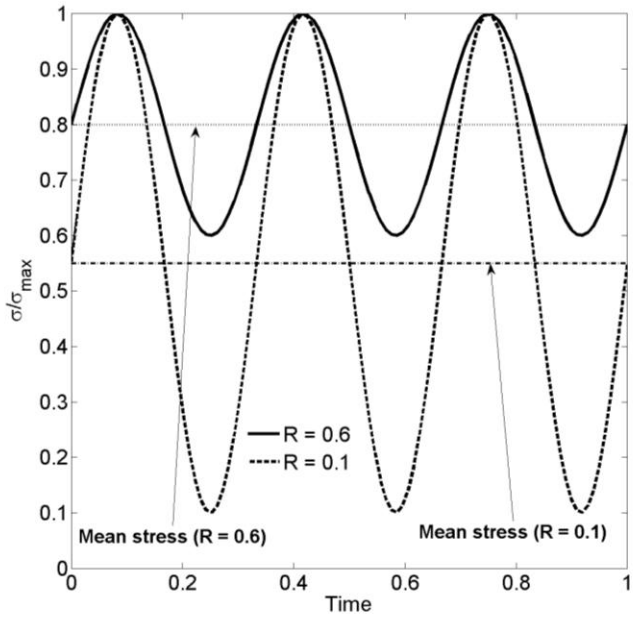

Figure 3. Two load histories with the same maximum stress but different values of R and mean stress.

(e) Fatigue Damage Accumulation in a Composite

The final piece of the composite fatigue puzzle is linking the rate of microcrack accumulation with macroscopic failure of the composite. Hansen and Baker-Jarvis (1990) linked KTF with macroscopic damage by introducing a damage parameter n that represented the percentage of microcrack density relative to the microcrack density at failure. The damage variable, which represents the fraction of microcrack density required for macroscopic fracture, is zero initially and is unity at failure. In their formulation, they introduced a differential equation for the evolution of a damage variable n with time t, where the evolution of the damage variable is directly related to the bond rupture rate. This method successfully predicted the strength of polymers subjected to a wide range of stress rates.

We use a differential equation describing the evolution of *n* that is similar to the one proposed by Hansen and Baker-Jarvis

$$\frac{dn}{dt} = (n_0 - n)^{\lambda} K_b, \quad n(0) = 0. \tag{9}$$

n_0 is a parameter that is determined by enforcing the condition

$$\int_0^1 \frac{dn}{(n_0 - n)^{\lambda}} = 1 \tag{10}$$

Combining Equations (9) and (5) and assuming a constant nominal temperature gives the starting equation for determining the fatigue life of a polymer.

$$\frac{dn}{dt} = (n_0 - n)^{\lambda} \frac{kT}{h} \exp\left(-\frac{U - \gamma\sigma(t)}{kT}\right), \quad n(0) = 0 \tag{11}$$

Solving Equation (11) yields the evolution of the damage parameter with time, which can be written as:

$$\begin{aligned} n(t) &= n_0\left(1 - \exp\left\{-\frac{kT}{h}\exp\left(\frac{-U}{kT}\right)\int_0^t \exp\left(\frac{\gamma\sigma_{eff}(\tau)}{kT}\right)d\tau\right\}\right), \quad \lambda = 1 \\ n(t) &= n_0 - \left\{n_0^{1-\lambda} - (1-\lambda)\frac{kT}{h}\exp\left(\frac{-U}{kT}\right)\int_0^t \exp\left(\frac{\gamma\sigma_{eff}(\tau)}{kT}\right)d\tau\right\}^{\frac{1}{1-\lambda}}, \quad \lambda \neq 1 \end{aligned} \tag{12}$$

In order to use Equation (12) to predict fatigue failure, the values of U and γ must be calibrated. This is accomplished by calibrating Equation (12) to a single off-axis S-N curve.

3: VALIDATION: PREDICTION OF SHORT BEAM SHEAR FATIGUE LIFE

To demonstrate the capability of this methodology, we use it to predict the fatigue life of a short beam shear specimen and compare these results with experimental results from Makeev et al. (2009). A finite element model of the short beam shear specimen was created in Abaqus according to the geometric descriptions described by Makeev et al. Figure 4 shows an illustration of this model. The FE model was run for applied loads of varying magnitude corresponding to peak loads of sinusoidal load histories with f = 10 Hz and R = 0.1. The results of the finite element analysis were used by fe-safe/Composites™, which embodies the methodology described in this paper, to predict fatigue life.

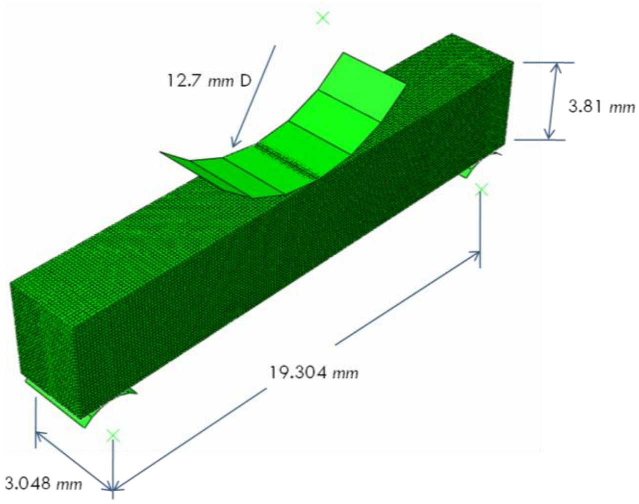

Figure 4: Illustration of the short beam shear finite element model.

The composite material modelled was the carbon/epoxy IM7/8552 material system. Composite properties for this system are given in Table 1 and come from the reported properties by Makeev et al (2009). Constituent properties, also shown in Table 1, come from previous analyses using similar materials. The tensile and compressive transverse composite strengths along with the in-plane shear strength are also required for the fatigue analysis. These values were taken to be: $^{+}S_{22}$ = 82.7 MPa, $^{-}S_{22}$ = 260.0 MPa (taken from a similar material), and S_{12} = 110.0 MPa. In addition to static properties, lamina fatigue calibration data is required to properly calibrate the parameters in the KTF equations. The interlaminar tension fatigue data presented by Makeev et al. (2009) were used to calibrate the model parameters.

Material	E_{11} (GPa)	$E_{22} = E_{33}$ (GPa)	$\nu_{12} = \nu_{13}$	ν_{23}	$G_{12} = G_{13}$ (GPa)
Composite	154	8.96	0.32	0.5	5.32
Fiber	268	15	0.23	0.2	49
Matrix	3.3	3.7	0.43	0.37	1.6

Table 1. Elastic material properties for the IM7/8552 composite and its constituents.

Using fe-safe/Composites™ in conjunction with Abaqus, contour plots of the fatigue life in the short beam shear tests were generated. The contour plot for a maximum stress of 55 MPa (calculated according to the method described by ASTM D 2344 (2006) is shown in Figure 5. The number of cycles to failure for the specimen was taken to be the lowest number of cycles along a vertical line equidistant between the center load point and the support (shown in Figure 5).

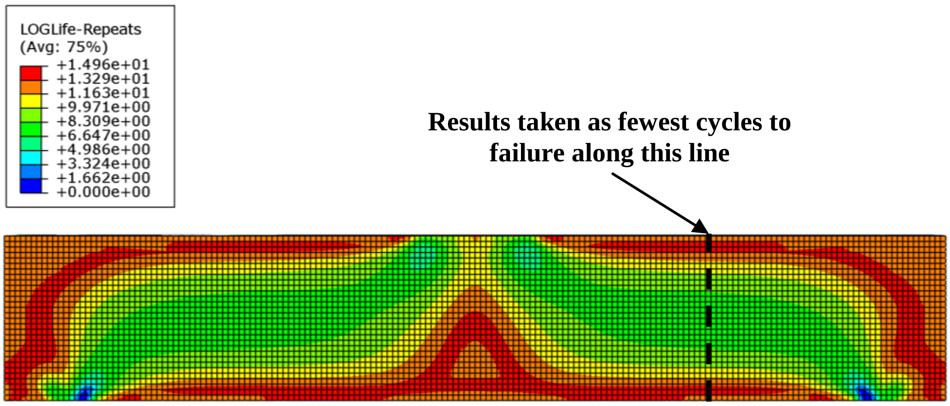

Figure 5: Contour of the number of cycles to failure for S_{max} = 55 MPa.

Figure 6 shows the predicted number cycles to failure computed for various peak loadings compared with the published data from Makeev et al. (2009). Although the model prediction is slightly conservative, the agreement is remarkable. *Tension* fatigue data was used calibrate the model; but the model was successfully used to predict *shear* behaviour. This excellent correlation with experiment suggests that the composite physics is being correctly modelled and opens the door to accurate fatigue prediction for larger composite structures.

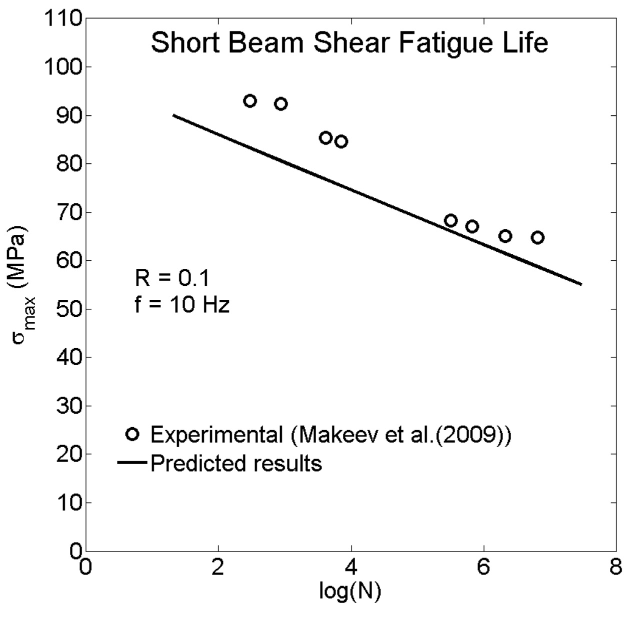

Figure 6: Comparison of predicted fatigue life with experimental values

4: SUMMARY AND CONCLUSIONS

We have outlined a physics-based model that is computationally efficient and is easily incorporated into finite element modelling. The framework presented in this paper is commercially packaged as Helius:Fatigue™ and is used to power the fe-safe/Composites™ module. The methodology utilizes multicontinuum theory to extract matrix stress in the composite from the composite stress state. The kinetic theory of fracture was chosen to model the appropriate physics of composite fatigue, where the matrix stress is used to drive the fatigue of the composite.

This methodology was used to predict the short beam shear fatigue life given only standard static composite properties and a single interlaminar tension curve. Excellent agreement was obtained between model results and experiment, suggesting that the applied physics is appropriate and that the method is suitable for fatigue modelling of larger composite structures.

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