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Predicting Composite Fatigue Life Using Constituent-Level Physics

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Use of composite materials is widespread in large aerospace structures. Many of these applications require the composite structure to perform under cyclic loading. Thus, fatigue life prediction in composite structures is an important part of composite design. In this paper, we present a comprehensive physics-based methodology for accurately predicting fatigue life in composite structures, which has been incorporated into the commercial software, Helius:Fatigue™. This methodology uses minimal coupon-level data for characterization (standard static tests plus two S-N fatigue curves). The basic framework for our approach is to use multicontinuum theory (MCT) to extract relevant constituent stresses from a composite stress or strain field and apply the kinetic theory of fracture (KTF) to predict fatigue life of the matrix constituent. Using KTF in conjunction with a damage variable allows the fatigue life of a composite to be accurately predicted. To demonstrate this approach, we evaluate the fatigue life of a composite plate with a hole in uniaxial tension fatigue.

Nomenclature

B_i	=	static failure coefficients of the matrix failure criterion
$\mathbf{C}_c$	=	composite stiffness matrix (6x6)
$\mathbf{C}_f$	=	fiber stiffness matrix (6x6)
$\mathbf{C}_m$	=	matrix stiffness matrix (6x6)
$\dot{E}$	=	rate of energy dissipation due to frictional heating
h	=	Planck's constant
$\mathbf{I}$	=	identity matrix
I_t	=	transversely isotropic matrix stress invariant corresponding to maximum matrix stress normal to the fiber
I_{s1}	=	transversely isotropic matrix stress invariant corresponding to in-plane matrix shear
I_{s2}	=	transversely isotropic matrix stress invariant corresponding to out-of-plane matrix shear
J''	=	loss compliance of the polymer matrix
k	=	Boltzmann constant
K_b	=	rate of microcrack accumulation and coalescence
n	=	fatigue damage variable
n_0	=	equilibration parameter that depends on the damage accumulation exponent
N_f	=	number of cycles to failure
N_i	=	number of cycles to initiate damage
N_p	=	number of cycles to propagate damage
R	=	fatigue load ratio
R_p	=	ratio of first two principal composite stresses
T	=	temperature driving the damage accumulation process
T^*	=	ambient temperature
U	=	activation energy associated with microcrack accumulation coalescence

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β	=	pressure strengthening coefficient
γ	=	activation volume associated with microcrack accumulations and coalescence
Δt	=	change in time over a particular segment of a load-history curve
ΔT	=	temperature change from a stress-free composite state
$\Delta \sigma_a$	=	change in stress over a particular segment of a load-history curve
$\boldsymbol{\eta}_c$	=	vector describing the coefficient of thermal expansion (CTE) of the composite
$\boldsymbol{\eta}_f$	=	vector describing the CTE of the fiber
$\boldsymbol{\eta}_m$	=	vector describing the CTE of the matrix
λ	=	damage accumulation exponent
$\boldsymbol{\sigma}_c$	=	volume average composite stress tensor
$\boldsymbol{\sigma}_f$	=	volume average fiber stress tensor
$\boldsymbol{\sigma}_m$	=	volume average matrix stress tensor
τ_0	=	bulk matrix shear strength
ϕ_f	=	fiber volume fraction
ϕ_m	=	matrix volume fraction
ψ	=	hysteresis heating parameter

I. Introduction

FOR polymer-matrix composites, fatigue failure is primarily a matrix-dominated event. Fatigue damage begins with microcrack accumulation in the polymer. These microcracks accumulate most rapidly in the early stages of fatigue life, with the accumulation rate slowing with increasing number of cycles. Ultimately, these microcracks nucleate a macroscopic crack that quickly causes catastrophic failure. Accurate fatigue life prediction thus requires modeling the accumulation of microcracks during cyclic loading. Ultimate catastrophic failures typically fall into one of three categories: off-axis failure, on-axis failure, and delamination. Off-axis failure occurs when composite tensile loading is more than a few degrees from the fiber axis[1]. The ultimate failure is characterized by matrix cracking parallel to the fiber[2,3] or by debonding between the fiber and matrix interface. On-axis failure occurs when a tensile fatigue load is applied in the direction of the fibers. As with the off-axis failure, microcracking occurs first in the matrix, but any matrix crack that may form cannot easily propagate through the fibers. Rather, cracks are bridged by the fibers, causing an increase in number and magnitude of stress concentration points in the fiber[4]. In the case of a composite laminate, the additional failure mode of delamination is possible. Again, delamination is a final event that results from accumulated matrix microcracking, thus accounting for the accumulation of matrix microcracks is sufficient for predicting fatigue failure due to delamination. Note that for all three damage categories, matrix cracking is the primary driver.

From a certain point of view, fatigue in composites and fatigue in metals are similar: both begin with damage initiation, followed by damage propagation, and end in ultimate failure. Fatigue life N_f, or number of cycles to failure, for both can then be thought of as the sum of the cycles during damage initiation N_i and the cycles during damage propagation N_p.

$$N_f = N_i + N_p . \tag{1}$$

But the difference is in the relative time spent in each phase. In the case of metals, a significant portion of the fatigue process is spent propagating a single crack. The initiation of damage is usually ignored because generally many defects such as grain boundaries and dislocations exist in the material that can replicate new defects. The propagation phase is longer because metals strain harden. As a crack attempts to propagate through the metal, plasticity occurs at the crack tip causing crack blunting and strain hardening. The process of crack blunting, strain hardening, and crack progress can be repeated for many thousands of cycles. So in the case of metals, Eq. (1) is simplified to

$$N_f = N_p . \tag{2}$$

The amount of crack growth during each cycle of the propagation phase is often described with an empirical law, such as the Paris Law.

In the case of a composite, such as a unidirectional carbon-epoxy laminate, strain hardening is negligible. This makes the propagation phase of the fatigue life much shorter than the damage initiation phase because once a defect of sufficient size is nucleated, damage progresses very quickly to ultimate failure. Thus, for fiber-reinforced polymer (FRP) composites, Eq. (1) is simplified to

$$N_f = N_i . \tag{3}$$

The initiation of damage in a FRP is governed by a kinetic process of microcrack accumulation and coalescence[5]. When a critical density of microcracks is achieved, a macroscopic crack forms and ultimate failure follows shortly thereafter. This type of fatigue failure can be modeled with the kinetic theory of fracture (KTF) [6-11]. In the case of a FRP composite material, the stresses in the polymer matrix are not the same as the composite stresses. In order to apply KTF to the polymer, a methodology for determining matrix stresses from composite level stresses must be implemented. In the methodology described, we use multicontinuum theory (MCT) to extract these polymer matrix stresses from composite stresses, as described below.

II. General Methodology

In the proposed methodology, composite fatigue is modeled as a matrix phenomenon using matrix-specific physics. This requires three separate modeling efforts: a multiscale modeling methodology to link composite stresses and strains to matrix stresses and strains, a physics-based model for fatigue of the matrix material, and a link between the physics-based model and the macroscopic fatigue failure of the matrix. A portion of these modeling efforts have been described by Fertig[12]. We emphasize that the goal of the methodology described is to predict mean fatigue behavior not the absolute probability of failure; rather the proposed method is intended to be used as a design tool to compare different layups, materials, and structural variations so that fatigue damage can be minimized in a particular structural design.

A. Multiscale Approach for Linking Composite Stress with Matrix Stress

The multiscale modeling methodology employed for linking composite stress/strain behavior with constituent stress/strain behavior is multicontinuum theory (MCT). The details of MCT are available in the open literature[13,14]. Briefly, MCT provides an exact mapping of volume average composite stresses/strains to volume average constituent stresses/strains. Thus any arbitrarily complex composite stress state can be mapped to its corresponding matrix stress state. This mapping is extremely computationally efficient, adding almost no computational burden to a finite element analysis of a composite structure. Given a composite stress state $\boldsymbol{\sigma}_c$, the matrix stress state $\boldsymbol{\sigma}_m$ can be simply computed as

$$\boldsymbol{\sigma}_m = \mathbf{Q}_m \boldsymbol{\sigma}_c - \boldsymbol{\lambda}_m \Delta T \tag{4}$$

where

$$\begin{aligned}
\mathbf{Q_m} &= \mathbf{C_m} \left\{ \mathbf{C_c} \left(\phi_m \mathbf{I} + \phi_f \mathbf{A} \right) \right\}^{-1} \\
\boldsymbol{\lambda}_\mathbf{m} &= \mathbf{C_m} \left\{ \phi_f \left[\left(\mathbf{C_c} - \mathbf{C_f} \right) \left(\phi_m \mathbf{I} + \phi_f \mathbf{A} \right) \right]^{-1} \mathbf{a} + \boldsymbol{\eta}_\mathbf{m} - \left(\phi_m \mathbf{I} + \phi_f \mathbf{A} \right)^{-1} \boldsymbol{\eta}_\mathbf{c} \right\} \\
\mathbf{A} &= -\frac{\phi_m}{\phi_f} \left(\mathbf{C_c} - \mathbf{C_f} \right)^{-1} \left(\mathbf{C_c} - \mathbf{C_m} \right) \\
\mathbf{a} &= \mathbf{C_c} \boldsymbol{\eta}_\mathbf{c} - \phi_f \mathbf{C_f} \boldsymbol{\eta}_\mathbf{f} - \phi_m \mathbf{C_m} \boldsymbol{\eta}_\mathbf{m}
\end{aligned} \tag{5}$$

and the stiffnesses $\mathbf{C}_i$ ($i = c$, f, or m) and thermal expansion coefficients $\boldsymbol{\eta}_i$ ($i = c$, f, or m) of the composite and constituents and the fiber volume fraction ϕ_f are the only properties needed to compute this mapping.

B. Kinetic Theory of Fracture (KTF)

The relevant physics describing the fatigue of a polymer matrix is captured by the kinetic theory of fracture (KTF) [6,7,15,8,9], which describes fracture as a thermally activated process. The basic premise for modeling thermally activated processes is illustrated in Fig. 1 (Left). In order for a system to transition from one thermodynamic state to another an energy barrier U, called the activation energy, must be overcome. Energy due to thermal oscillation of molecules conforms to a Maxwell-Boltzmann distribution such that a likelihood, given by $\exp\left(-\frac{U}{kT}\right)$, always exists that a thermal oscillation will overcome the energy barrier and move to a new state, where k is the Boltzmann constant and T is the temperature at which the transition is taking place. The frequency at which an attempt is made to cross this energy barrier is roughly equal to kT/h, where h is Planck's constant. The rate at which this transition takes place is given by the product of the attempt frequency and the likelihood of success.

In the case of fracture or fatigue of a material, we are interested in the transition from a state of unbroken bonds (no microcracks) to broken bonds (microcracked). An applied stress σ can serve to reduce the apparent activation energy, as shown in Fig. 1 (Right), such that the apparent activation energy barrier is reduced to $U - \gamma\sigma$. γ is defined as an activation volume that corresponds roughly to the volume of material participating in the activated process. Using this idea, the most basic form of the KTF equation proposed by Zhurkov [10] and Tobolsky and Eyring [16,17] can be modified to give the rate K_b of thermally activated transition from an unmicrocracked state to a microcracked state.

$$K_b = \frac{kT}{h}\exp\left(-\frac{U-\gamma\sigma}{kT}\right) \tag{6}$$

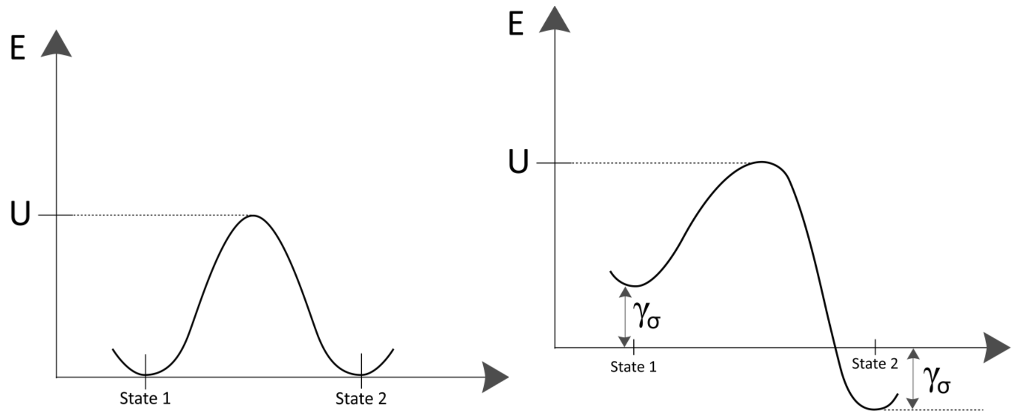

Figure 1. (Left) Energy barrier with activation energy *U* for transition from State 1 to State 2. (Right) Energy barrier with activation energy *U-γσ* for transition from State 1 to State 2.

Eq. (6) gives the rate of microcracking under constant stress and temperature. But studies on fatigue in polymers have shown the potential for a significant increase in temperature due hysteresis heating[15,8,18]. This heating will be dependent on both the magnitude of the stress oscillation and the frequency of the oscillation. While our methodology does not explicity model the viscoelastic component of fatigue, we include a portion of the viscoelastic effects via a modification of the temperature at which the microcracking process occurs. Sauer and Richardson[8] have shown that the rate of energy dissipated is given by

$$\dot{E} = \pi f J'' \sigma_a^2 \tag{7}$$

where f is the oscillation frequency, J'' is the loss compliance of the polymer, and σ_a is the amplitude of the stress oscillation. Eq. (4) is given for a sinusoidal load history. To extend this idea to an arbitrary load history discretized into q segments we assume that temperature increase is proportional to the energy dissipation rate. Thus, the temperature T at which microcracking occurs is related to the ambient temperature T^* by

$$T = T^* + \psi \sum_{i=1}^{q} \frac{(\Delta\sigma_a)^2}{\Delta t} \tag{8}$$

where $\Delta\sigma_a$ is the change in stress over a particular segment of the load history, Δt is the time required to complete the segement, and ψ is a material parameter describing the effect of hysteresis heating on the polymer. The incorporation of heating as a function of stress amplitude enables this methodology to accurately model the effect of mean stress on composite fatigue life, which has been well described by Kawai and Suda[19]. Substituting Eq. (8) into Eq. (9) and allowing for a time-dependent stress yields the KTF equation that will be used for predicting the rate of microcrack accumulation and coalescence in the matrix of a polymer matrix composite.

$$K_b = \frac{k\left(T^* + \psi \sum_{i=1}^{q} \frac{(\Delta\sigma_a)^2}{\Delta t}\right)}{h} \exp\left(- \frac{U - \gamma\sigma(t)}{k\left(T^* + \psi \sum_{i=1}^{q} \frac{(\Delta\sigma_a)^2}{\Delta t}\right)} \right) \tag{9}$$

C. Linking physics with macroscopic behavior

1. Tracking damage in the composite

In order to utilize Eq. (9) in a macroscopic model, it must be linked with a macroscopic measurement of damage. We follow the work of Hansen and Baker-Jarvis[20], who used KTF to investigate effects of load rate on the behavior of bulk polymers, and use a damage parameter *n* that describes microcrack density accumulated as a percentage of the microcrack density at failure[‡]. We define the damage variable such that before any fatigue damage, $n = 0$; fatigue failure is considered to occur when $n = 1$. We assume that the rate of damage accumulation *n* will be proportional to the rate of microcracking as defined in Eq. (9), and introduce a differential equation, similar to the one proposed by Hansen and Baker-Jarvis, to describe the evolution of *n*

$$\frac{dn}{dt} = (n_0 - n)^{\lambda} K_b, \quad n(0) = 0 \tag{10}$$

where λ is a damage accumulation exponent, K_b is given by Eq. (9), and n_0 is an equilibration parameter that forces Eq. (10) to yield the creep solution under constant load; it is determined by enforcing the condition

$$\int_0^1 \frac{dn}{(n_0 - n)^{\lambda}} = 1 \tag{11}$$

Eq. (8) can be solved numerically for any arbitrary load history.

We have noted above that two types of damage are of particular interest: matrix cracks that form parallel to the fibers, which control off-axis fatigue; and matrix cracks that form perpendicular to the fibers, which control on-axis fatigue by progressively increasing the load carried by the fibers. To account for both of these modes of damage we assign a unique damage variable to *each* damage mode. This effectively accounts for the tensorial nature of damage in a real composite. Each damage variable is evolved independently by directly integrating Eq. (10). The treatment of multiple damage modes necessitates that each be assigned its own activation volume, activation energy, and hysteresis heating parameter. Thus, we predict a fatigue life associated with each damage mode.

Hashin and Rotem[1] showed that for uniaxial in-plane loading, longitudinal failures only occur when the loading axis is less than 2° from the fiber axis; otherwise transverse failure occur. However, because the composite stress states generated in practice will rarely be purely in-plane or uniaxial, we have developed a method based on the results of Hashin and Rotem for determining the *mode* of fatigue failure under arbitrary composite loading. The method uses the following steps to determine the failure mode of the composite.

a. The principal stresses (σ_I, σ_{II}, and σ_{III}) in the composite are calculated along with their corresponding directions, where $\sigma_I \geq \sigma_{II} \geq \sigma_{III}$.

b. If σ_I is negative or its direction is more than 2° away from the fiber axis, transverse failure is predicted.

c. If σ_I is positive and its direction is less than 2° away from the fiber axis, the ratio between the first and second principal stresses $R_P = \frac{\sigma_I}{\sigma_{II}}$ is calculated. If $R_P > 50$, longitudinal failure is enforced. Otherwise the fatigue life values for both transverse and longitudinal failure modes are computed; the failure mode that yields the shortest fatigue life is selected as the failure mode.

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‡ Kozin and Bogdanoff [21] have also demonstrated the link between KTF in metals and incremental crack growth introducing the crack length *a* as a damage parameter.

2. Calculation of an effective matrix stress

In conjuction with finite element analysis, the use of Eq. (4) yields a volume average matrix stress *tensor* at each integration point. However, in order to use this stress in conjuction with KTF a *scalar* measure of the matrix stress is required. Multiple researchers have shown that fatigue stress normalized by the static failure strength is an appropriate parameter for predicting fatigue life[2,22,23]. Consequently, we use the functional form for a static failure criterion to determine an effective stress. This is essentially equivalent to the strength-life equal rank assumption[24] (SLERA). One weakness, however, to blindly applying SLERA is that it fails in the longitudinal directions. Fatigue due to longitudinal loading is dependent on the accumulation of matrix microcracking, but static strength is completely governed by fiber strength. Thus, in order to treat PMC fatigue as a matrix phenomenon using KTF, we compute an effective stress for *each* fatigue failure mode based on the stresses that would drive static failure for that mode.

We first consider the off-axis failure mode. For this mode, it has been widely reported that in unidirectional composites fatigue failure often occurs via cracking parallel to fibers[3,25]. Thus, it is expected that tensile forces perpendicular to the fibers will play a substantial role in fatigue as well as the shear stresses on these planes. Taking the axial direction of the fiber to be the 1-direction, we propose a matrix failure criterion in the form of transversely isotropic invariants of the *matrix stress tensor* calculated using MCT that takes the form

$$B_t \{I_t\}^2 + \frac{1}{\left(1 - \frac{\beta}{\tau_0}\{-I_h\}\right)}\left[B_{s1} I_{s1} + B_{s2} I_{s2}\right] = 1 \tag{12}$$

where

$$\begin{aligned} I_t &= \frac{\sigma_{22}^m + \sigma_{33}^m + \sqrt{\left(\sigma_{22}^m + \sigma_{33}^m\right)^2 - 4\left(\sigma_{22}^m \sigma_{33}^m + \left(\sigma_{23}^m\right)^2\right)}}{2} \\ I_{s1} &= \left(\sigma_{12}^m\right)^2 + \left(\sigma_{13}^m\right)^2 \\ I_{s2} &= \tfrac{1}{4}\left(\sigma_{22}^m - \sigma_{33}^m\right)^2 + \left(\sigma_{23}^m\right)^2 \\ I_h &= \sigma_{22}^m + \sigma_{33}^m \end{aligned} \tag{13}$$

the {} denote Macaulay brackets, such that the value is zero if the encompassed quantity is negative, and σ^m_{ij} correspond to components of the volume average matrix stress tensor. The values of B_i are determined from three lamina-level static failure tests: transverse tension, transverse compression, and in-plane shear, all of which involve failure of the matrix constituent. β is a pressure strengthening coefficient and τ_0 is the matrix shear strength with no pressure strengthening. In the current version, all calculations use $\beta = 0$.The accuracy of this failure criterion has been demonstrated for off-axis static failure elsewhere[12].

Eq. (12) can be used to obtain an off-axis effective stress by simply dividing by the failure coefficient B_{s1} to obtain

$$\sigma_{eff}^{off-axis} = \sqrt{A_t \{I_{m,t}\}^2 + \sigma_{m,12}^2 + \sigma_{m,13}^2 + A_S\left(\tfrac{1}{4}\left(\sigma_{m,22} - \sigma_{m,33}\right)^2 + \sigma_{m,23}^2\right)} \tag{14}$$

where the coefficients in the effective stress can be readily obtained from the static failure coefficients as

$$A_t = \frac{B_t}{B_{s1}}, \quad A_S = \frac{B_{s2}}{B_{s1}} \tag{15}$$

The effective stress for on-axis loading is taken to be simply the tensile matrix stress normal to the fibers

$$\sigma_{eff}^{on-axis} = \{\sigma_{11}^m\} \tag{16}$$

D. Integration with Finite Element Analysis

The methodology described in sections II.A-II.C has been developed commercially as Helius:Fatigue™, which has recently been integrated into the fatigue modeling software fe-safe/Composites™ [26]. Currently this software is designed to be used as a post-processor for finite element modeling results. Fatigue analysis of a composite structure proceeds as follows. First, a material must be calibrated for use with the methodology described; this includes computing *in situ* composite properties as well as KTF parameters U, γ, and ψ. The calibration of these parameters requires elastic material properties, static strength data, and two lamina-level fatigue curves (a longitudinal faituge curve and an off-axis fatigue curve). After the material is characterized, a finite element model is created with a reference load P_{ref}, typically corresponding to the largest load in the load history. The finite element model is then executed, which generates an output file. fe-safe/Composites™ reads composite stress information from the output file and sends composite stresses to the Helius:Fatigue™ module which performs the methodology described in this paper. The software returns the number of cycles to failure and the failure mode (longitudinal or transverse) to the finite element output file for viewing using the finite element software.

III. Simulation and Results

We have shown elsewhere that the methodology proposed here works well for off-axis fatigue life prediction of unidirectional composite laminae[12]. Here we investigate the ability of the model to predict angle-ply *laminate* behavior based on *lamina* characterization. Unfortunately, most of the published laminate fatigue data does not provide any data with which to characterize the laminate. The work by Rotem and Nelson[27] is one of the few that provides adequate lamina data, in this case for the T300-5208 carbon/epoxy system at room temperature. We used this data to characterize the material. The properties used are given in Table 1 (units of stress are given in MPa). The values with a * correspond to assumed values taken from similar carbon/epoxy materials[28] Fatigue curves given for 0° loading and 90° loading were used to characterized the on-axis and off-axis fatigue failure modes, respectively.

Table 1. T300/5208 Carbon-Epoxy Lamina Properties

	E_{11}	E_{22}	G_{12}	ν_{12}	ν_{23}	$+S_{22}$	$-S_{22}$	S_{12}
Composite	136000	8729	6000	0.36	0.4*	44	200*	74
T300 Carbon*	207000	11382	24226	0.33	0.21			
5208 Epoxy*	4556	4556	1631	0.4	0.4			

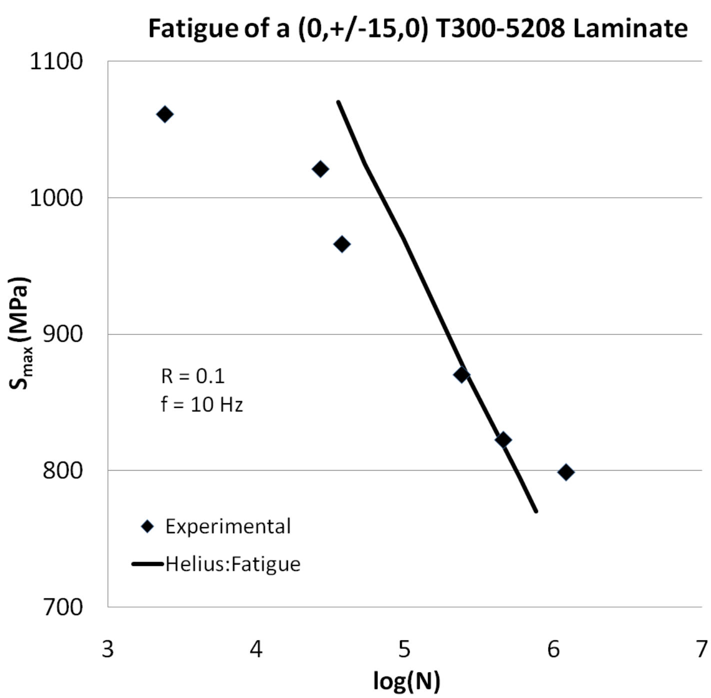

Figure 2. Comparison of Helius:Fatigue predictions for (0, ±15, 0)$_S$ carbon/epoxy laminate (with the experimental results reported by Rotem and Nelson[27].

Using the characterized lamina properties, we used Helius:Fatigue™ packaged as fe-safe/Composites™ to predict the fatigue life of a symmetric angle-ply laminate (0, ±15, 0)$_S$. Fig. 2 shows the predicted fatigue life compared with the experimental results reported by Rotem and Nelson[27]. Our results predict first-ply fatigue failure. In this case, the 15° plies fail first in an off-axis mode. Generally, the agreement between the predicted results and the experimental results was remarkable. And we stress again that we have used *lamina* fatigue data to predict *laminate* fatigue life. The deviation from the experimental curve at the highest stress reported is believed to be because at high stresses, significant damage may occur during a single loading curve, thus creating discrete damage events such that the assumption of no damage before the completion of the first cycle may not be valid. Additionally, nonlinear behavior of the matrix may play a more important role at higher stresses—to capture this behavior a progressive fatigue failure solution should be implemented.

The above results demonstrate good agreement with coupon-level data. But to demonstrate the extensibility of our methodology to structural-level FEA analysis, we also modeled open hole tension fatigue (R = 0.1, f = 10 Hz) for three different laminates: a soft laminate $(90, \pm 45, 90)_S$, a hard laminate $(0, \pm 45, 0)_S$, and a quasi-isotropic laminate $(0, \pm 45, 90)_S$. We use the properties of the carbon-epoxy system AS4/PEEK as reported by Kawai *et al.*[29]. No experimental data was available for comparison; the purpose of these results is to give the demonstrate the type of results produced by the methodology described here.

Figures 3-5 show the fatigue life contours (top) and failure modes (bottom) for the quasi-isotropic, soft, and hard laminates, respectively. (Because each of the ±45 plies have nearly identical fatigue life contours and failure modes, only the +45 ply is shown.) The fatigue life is given in terms of the logarithm of the cycles to failure. Failure modes correspond to: 1= longitudinal failure mode and 2 = transverse failure mode. Note that both 0° and 90° plies show can show both types of failure modes.

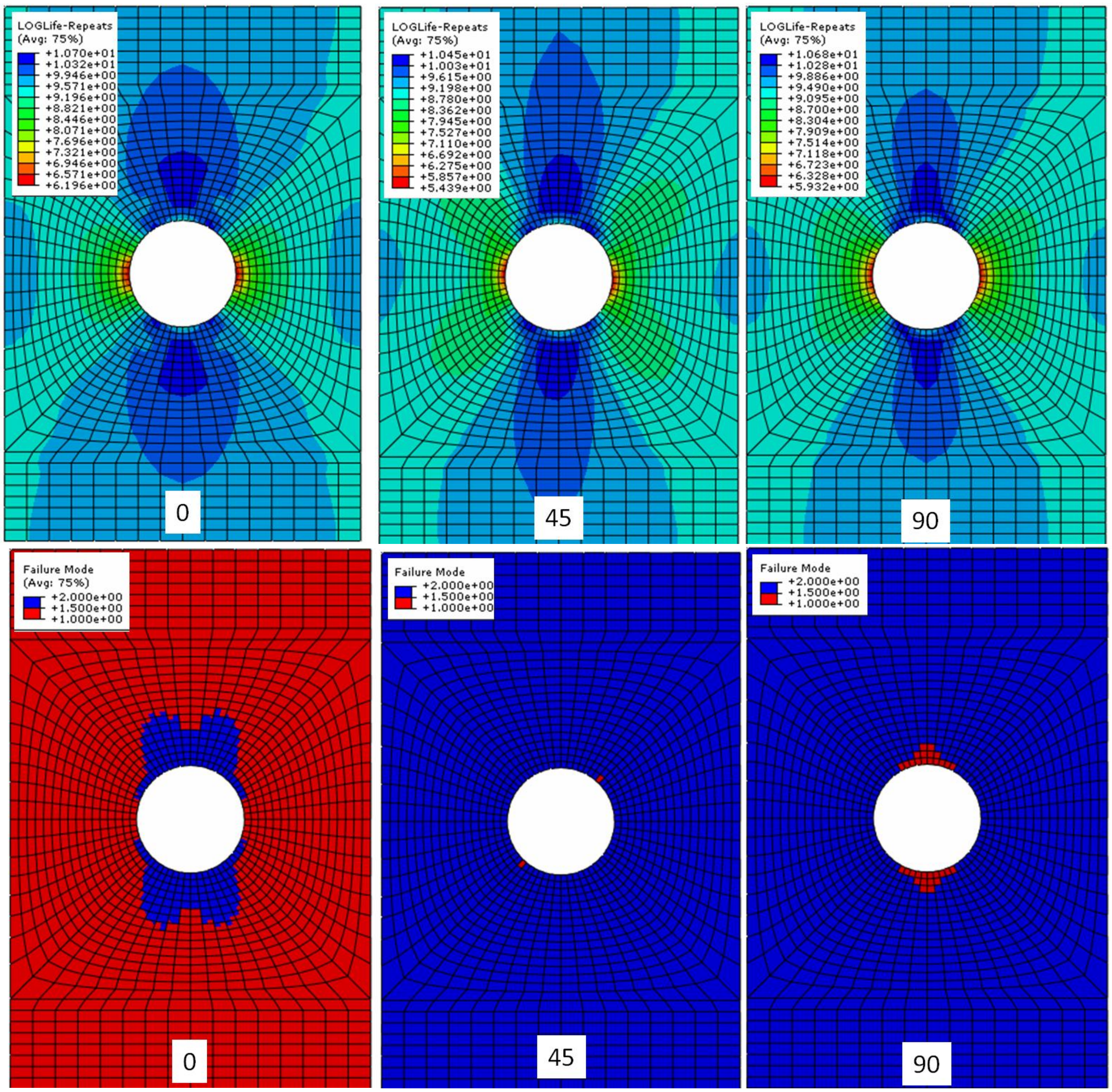

Figure 3. Initial fatigue failure in a quasi-isotropic laminate $(0/+45/-45/90)_S$

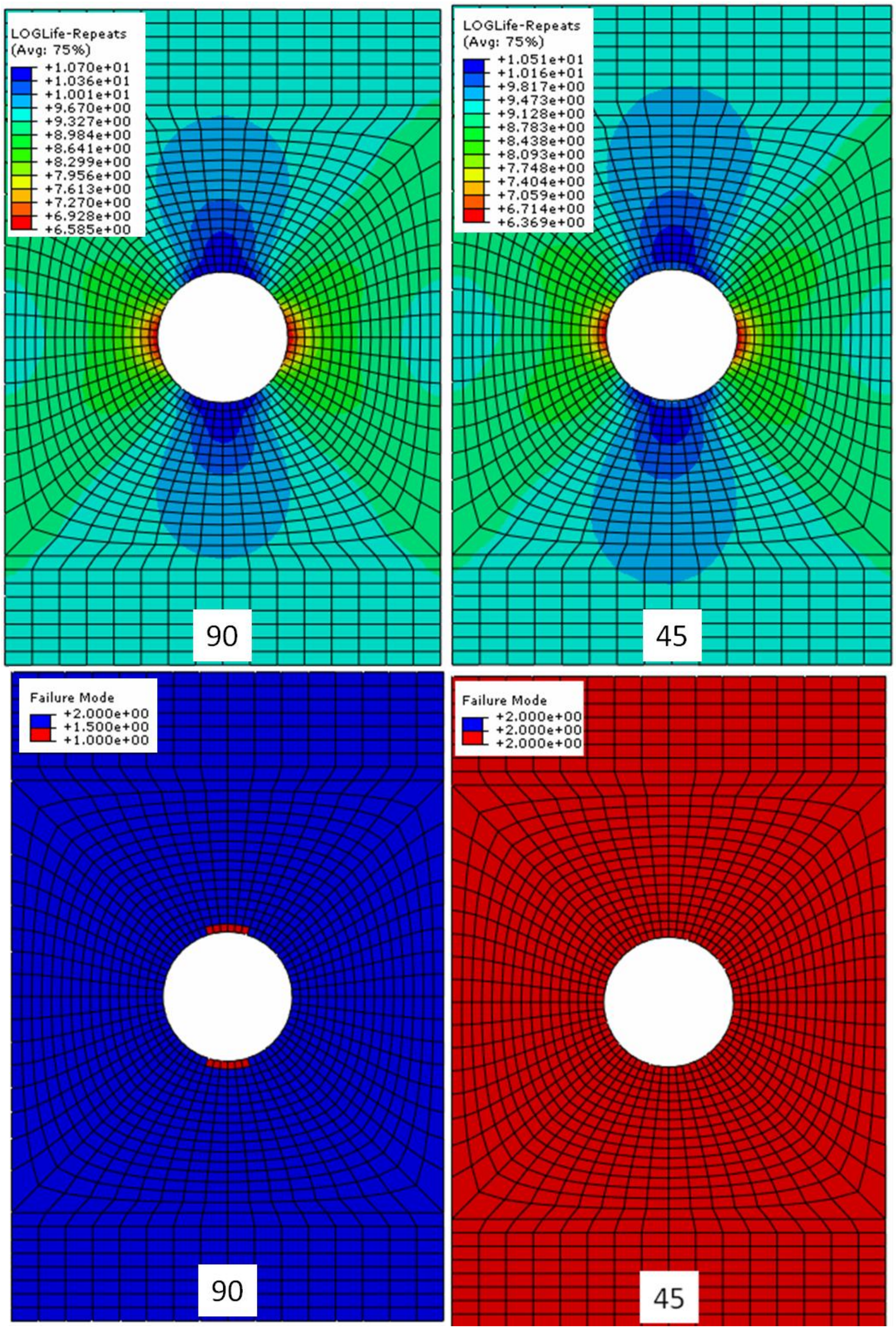

Figure 4. Initial fatigue failure in a soft laminate $(90/+45/-45/90)_S$

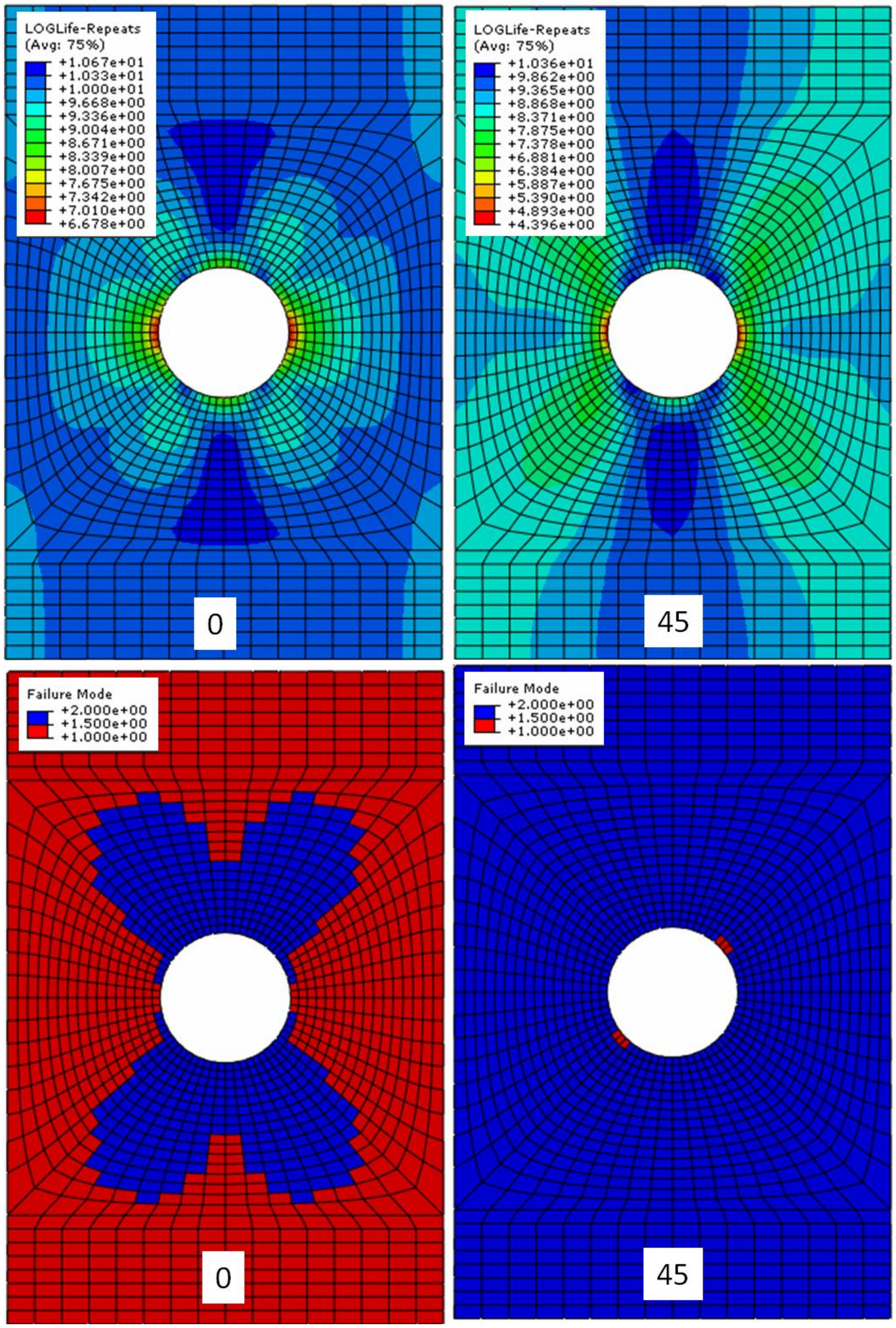

Figure 5. Initial fatigue failure in a hard laminate $(0/+45/-45/0)_S$

IV. Conclusion

We have outlined a methodology for predicting fatigue life of large composite structures. This methodology requires minimal material testing for characterization and is very computationally efficient. The approach is multi-scale and physics-based: multicontinuum theory is used to extract continuum stresses from composites stresses and the kinetic theory of fracture is used to predict fatigue in the matrix constituent by evolving multiple damage variables corresponding to multiple failure modes. In this paper we have shown the ability of the methodology to predict laminate fatigue data from lamina properties as well as the ability to predict fatigue life and multiple failure modes in a structural analysis. This approach and its implementation represent a game-changing step towards developing theoretically sound composite design tools that can readily be used by design engineers.

Acknowledgments

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